Simulating Astronomical Instrumentation
Constants, Formulae and Other (Hopefully!?) Useful Stuff

Mattia Vaccari
Department of Astronomy
University of Padova
Italy

mattia@mattiavaccari.net
http://www.mattiavaccari.net

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Abstract

Constants, formulae and other stuff (hopefully!?) useful in simulating the behaviour and performance to be expected from astronomical instrumentation is collected.

Introduction

Simulating the behaviour and performance of instrumentation is required, in different forms, at several different times during the development of an astronomical research project, from preliminary studies of an instrument to the planning of its observations to the assessment of their quality.
This is a compilation of mostly trivial and boring but (hopefully!?) useful stuff related to the simulation of astronomical instrumentation behaviour and performance.
In Section 1 some basic optical concepts are reviewed. Section 2 discusses energies involved in light propagation, its breaking down in photons and blackbody emission. In Section 3 reviews some typical representations of the spectrum of astronomical objects. Sections 4 and 5 discuss sources of instrumental and sky background, respectively. Section 6 reports a few formulae which can be used to compute the SNR to be expected from an observation. In Appendices A and B, the units of measure used for angular

and photometric quantities, the adopted conventions about their abbreviations, some useful conversion factors and formulae are given. Appendix D lists further useful astrophysical constants and conversion factors.
Léna *et al.* (1998), Schlessinger (1995), and Vaccari (2000) are good starting points for the adoption of a consistent set of units of measures, names and symbols.
SI units are generally used throughout the document. Approximate numerical values of constants (generally given with 4 significant figures) and analytical relations are indicated by the $\simeq$ symbol, whereas the $=$ symbol is used to indicate exact ("by definition") relations and physical constants.

1 Optics

The *Aperture* D of an optical system is (for radial-symmetry, i.e. circular, telescopes, which we will limit ourselves to consider) the diameter of its light-collecting area (i.e. its primary mirror or lens).
The *Focal Length* f of an optical system is expressed in metres and defines the image scale on the focal plane, or *Focal Scale* of the instrument, meaning that the angular distance $\Delta\,\alpha$ between two directions on the sky is related to their physical distance $\Delta\,l$ when they are imaged on the focal plane by

$$\Delta\,\alpha_{[\mathrm{rad}]} = \frac{\Delta\,l}{f} \ ,$$

which once "put in numbers" becomes

$$\frac{\Delta\,l_{[\mu\mathrm{m}]}}{\Delta\,\alpha_{[\mathrm{arcsec}]}} \simeq 4.848\,f_{[\mathrm{m}]} \ .$$

Similarly, the solid angle $\Delta\,\Omega$ spanned on the sky by a given sky region is related to the physical area $\Delta\,A$ spanned on the focal plane by

$$\Delta\,\Omega_{[\mathrm{sterad}]} = \frac{\Delta\,A}{f^2} \ ,$$

which once "put in numbers" becomes

$$\frac{\Delta\,A_{[\mathrm{m}^2]}}{\Delta\,\Omega_{[\mathrm{deg}^2]}} \simeq 3.046 \cdot 10^{-4}\,f^2_{[\mathrm{m}]} \ .$$

The *Focal Ratio* (also referred to as *f-number*, *f-ratio*, *Aperture Ratio* and *Relative Aperture* and indicated as F or N) is defined as the ratio between the focal length and the aperture, e.g. by

$$F = N = \frac{f}{D} \ .$$

It is often useful, and particularly so for simulation purposes, to describe the entire optical system of an astronomical instrument by means of a more or less simple mathematical function depending on a few parameters. In this context, it is also customary to describe the optical response of most "conventional" telescopes by means of some measure of the "size" of the instrument's beam.
A functional form often used for these purposes is the ideal diffraction pattern produced by a circular aperture, or *Airy Function*, whose exact analytical expression can be written as

$$AF(\phi) = 4\left[\frac{J_1(\phi) - \epsilon\, J_1(\epsilon\,\phi)}{\phi\,(1-\epsilon^2)}\right]^2 \tag{1}$$

where ϵ is the telescope obstruction ratio ($\epsilon = 0$ in ideal systems), J_1 is the Bessel J-function of order 1 and ϕ is the dimensionless parameter related to the sky angle α by

$$\phi = \pi\,\frac{D}{\lambda}\alpha \quad \text{[rad]}$$

For an ideal ($\epsilon = 0$) system the Airy Function thus reduces to

$$AF(\phi) = 4\left(\frac{J_1(\phi)}{\phi}\right)^2 \tag{2}$$

and its Encircled Energy Function to

$$EEF(\phi) = [1 - J_0^2(\phi) - J_1^2(\phi)] \tag{3}$$

The Airy Function and its Encircled Energy Function for an ideal system are tabulated in Table 1 for various values of α (here expressed in units of λ/D).

As mentioned, in practice a few "fiducial" values are used to characterize the extension of such an ideal diffraction pattern. An often-used proxy for an instrument's beam size is the *Airy Disk*, i.e. the first zero-brightness circle of the Airy Function. The angular diameter of the Airy Disk produced by a circular aperture of linear diameter D is

$$\delta_{AD} \simeq 2.44\,\frac{\lambda}{D} \quad \text{[rad]} \qquad \text{or} \qquad \delta_{AD} \simeq 0.503\,\frac{\lambda_{[\mu\mathrm{m}]}}{D_{[\mathrm{m}]}} \quad \text{[arcsec]}\,,$$

so that the overall size of the "beam", or the solid angle encircled by the

Table 1: Values of the Airy Function and its Encircled Energy Function as from Equations 2 and 3.

α [λ/D]	$AF(\alpha)$	$EEF(\alpha)$
0.0	1.000	0.0000
0.1	0.976	0.0244
0.2	0.905	0.0940
0.3	0.797	0.1989
0.4	0.664	0.3248
0.5	0.521	0.4559
0.6	0.381	0.5774
0.7	0.256	0.6785
0.8	0.154	0.7531
0.9	8.03E-2	0.8011
1.0	3.28E-2	0.8264
1.1	8.14E-3	0.8361
1.2	1.77E-4	0.8378
1.3	2.27E-3	0.8382
1.4	8.45E-3	0.8417
1.5	1.42E-2	0.8500

Airy Disk, is

$$\Omega_{AD} \simeq 4.68 \left(\frac{\lambda}{D}\right)^2 \quad \text{[sterad]} \qquad \text{or} \qquad \Omega_{AD} \simeq 0.199 \left(\frac{\lambda_{[\mu\text{m}]}}{D_{[\text{m}]}}\right)^2 \quad [\text{arcsec}^2]\ .$$

Now the "full" Airy Disk above is a rather conservative (i.e. pessimistic) measure of the "size"' of an instrument's beam, which is why one often uses the Airy Disk FWHM (or more precisely the FWHM of the Airy Fucntion) for this purpose instead, whose value is

$$\delta_{AD,FWHM} \simeq 1.02\, \frac{\lambda}{D} \quad \text{[rad]} \qquad \text{or} \qquad \delta_{AD,FWHM} \simeq 0.210\, \frac{\lambda_{[\mu\text{m}]}}{D_{[\text{m}]}} \quad \text{[arcsec]}\ ,$$

so that the overall size of the "beam", or the solid angle encircled by the Airy Disk FWHM, is

$$\Omega_{AD,FWHM} \simeq 0.817 \left(\frac{\lambda}{D}\right)^2 \quad \text{[sterad]} \qquad \text{or} \qquad \Omega_{AD,FWHM} \simeq 0.0348 \left(\frac{\lambda_{[\mu\text{m}]}}{D_{[\text{m}]}}\right)^2 \quad [\text{arcsec}^2]\ .$$

The *Focal Ratio* f/D of an optical system is the ratio between its focal length and aperture, whereas its reciprocal D/f is called *Aperture Ratio*.
For an ideal optical system (100 % reflectivity mirrors), the specific power P_ν hitting the whole detector when the system is exposed to a point source

of specific brightness F_ν is given by[1]

$$P_\nu = \frac{\pi\, D^2}{4}\, F_\nu \quad [\mathrm{J\ /\ s\,Hz}]\ , \tag{4}$$

or equivalently, expressed in photons, by

$$P_{\nu,ph} = \frac{\pi\, D^2}{4\, h}\, \frac{F_\nu}{\nu} \overset{25}{=} \frac{\pi\, D^2}{4\, h}\, \frac{F_\lambda}{\lambda} \quad [\mathrm{photons\ /\ s\,Hz}]\ . \tag{5}$$

Similarly, for an ideal optical system the specific power per unit solid angle hitting the detector when the system is exposed to a diffuse source of specific surface brightness Σ_ν is given by

$$P_{\nu,sur} = \frac{\pi\, D^2}{4}\, \Sigma_\nu \quad [\mathrm{J\ /\ s\,sr\,Hz}]\ , \tag{6}$$

or equivalently, expressed in photons, by

$$P_{\nu,ph,sur} = \frac{\pi\, D^2}{4\, h}\, \frac{\Sigma_\nu}{\nu} \overset{26}{=} \frac{\pi\, D^2}{4\, h}\, \frac{\Sigma_\lambda}{\lambda} \quad [\mathrm{photons\ /\ s\,sr\,Hz}]\ . \tag{7}$$

2 Energy, Photons, Blackbodies etc.

The energy of a photon of frequency ν is given by

$$E = h\ \nu = \frac{h\ c}{\lambda}\ ,$$

where

$$h \simeq 6.626 \cdot 10^{-34}\ \mathrm{J\ /\ s}\ ,$$

is Planck's constant and

$$c \simeq 2.999 \cdot 10^{8}\ \mathrm{m\ /\ s}\ ,$$

is the speed of light in a vacuum.
In spectroscopy, the energy of a photon is often expressed by the inverse of

[1]In Equations 4, 5, 6 and 7, a generic area A must be used instead of $\pi\, D^2/4$ for non-circular telescopes.

its wavelength, or *Wave Number*

$$\tilde{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} \ ,$$

which is generally expressed in cm^{-1}. When using an actual energy unit is appropriate, one generally uses the electronvolt

$$1 \ \mathrm{eV} = 1.602 \cdot 10^{-19} \mathrm{J} \ ,$$

i.e. the amount of energy equivalent to that gained by a single unbound electron when it is accelerated through an electrostatic potential difference of one volt (in a vacuum). In other words, an electronvolt is equal to one volt (1 volt = 1 joule per coulomb) multiplied by the (unsigned) charge of a single electron. The energy of a photon is related to its wavelength by the following

$$E_{[\mathrm{eV}]} = \frac{1.24}{\lambda_{[\mu\mathrm{m}]}} \ .$$

Planck's law describing the radiation from a blackbody says that the frequency specific surface brightness, i.e. the energy emitted per unit time, unit area (of the emitter's surface), unit solid angle and unit frequency interval, of a blackbody at a temperature T is

$$B_\nu(\nu, T) = \frac{2\,h\,\nu^3}{c^2} \left[\exp\left(\frac{h\nu}{kT} \right) - 1 \right]^{-1} \quad [\mathrm{J} \,/\, \mathrm{s\,m^2\,sr\,Hz}] \ ,$$

whereas the corresponding wavelength specific surface brightness, i.e. the energy emitted per unit time, unit area (of the emitter's surface), unit solid angle and unit wavelength interval (which is related to the previous quantity by Equation 24) is

$$B_\lambda(\lambda, T) = \frac{\nu^2}{c}\, B_\nu(\nu, T) = \frac{2\,h\,c^2}{\lambda^5} \left[\exp\left(\frac{hc}{\lambda kT} \right) - 1 \right]^{-1} \quad [\mathrm{J} \,/\, \mathrm{s\,m^2\,sr\,m}] \ ,$$

where

$$k \simeq 1.38 \cdot 10^{-23} \mathrm{J} \,/\, \mathrm{K} \ ,$$

is Boltzmann's constant.
The corresponding specific brightness at the receiver, i.e. the energy received per unit time, per unit area (of the receiver's surface) per unit frequency/wavelength interval from the whole blackbody (assuming this has a

spherical shape) is instead

$$\mathcal{SB}_{\nu\,(\lambda)} = \pi\ B_{\nu\,(\lambda)}\ \left(\frac{R}{r}\right)^2\ ,$$

where R is the radius of the blackbody, r the distance between the blackbody and the observer, and the π factor arises from integration.
Integration of one of the aforementioned formulae yields the energy emitted per unit time per unit area (of the emitter's surface) at all frequencies/wavelengths. This turns out to depend on the temperature of the blackbody only, a result also known as Stefan-Boltzmann's law

$$F_{bb} = \sigma\, T^4 \quad [\mathrm{J\,/\,s\,m^2}]\ ,$$

where

$$\sigma = \frac{2\,\pi^5}{15}\,\frac{k^4}{c^2\,h^3} \simeq 5.67 \cdot 10^{-8}\ \mathrm{J\,/\,s\,m^2\,K^4}$$

is Stefan-Boltzmann's constant.
The corresponding brightness at the receiver, i.e. the energy received per unit time, per unit area (of the receiver's surface) at all frequencies/wavelengths from the whole blackbody (assuming this has a spherical shape) is instead

$$\mathcal{B} = \pi\ F_{bb}\ \left(\frac{R}{r}\right)^2\ ,$$

where R is the radius of the blackbody, r the distance between the blackbody and the observer, and the π factor arises from integration.
Similarly, the derivation of the expression given for B_λ yields the relation between the wavelength of its maximum and the temperature, or Wien's displacement law

$$\lambda_{max}\, T \simeq 2.898 \cdot 10^{-3}\,\mathrm{m\,K}\ .$$

The *Emissivity* ε of a body is defined as the ratio

$$\varepsilon_\lambda = \frac{F_{true,\lambda}}{F_{bb,\lambda,T}}\ ,$$

between its wavelength specific brightness and the wavelength specific brightness of a blackbody at the same temperature T. The *Absorptivity* of a body is similarly defined as the ratio of the energy absorbed by a body and by a blackbody at the same temperature. *Kirchoff's law* states that at thermal equilibrium, the emissivity of a body equals its absorptivity. In general, the

emissivity of a body is wavelength-dependent, but by definition a blackbody has got $\varepsilon_\lambda \equiv \varepsilon$. By extension, a graybody is a body for which $\varepsilon_\lambda \equiv \varepsilon$, and thus $F_{true,\lambda} = \varepsilon \; F_{bb,\lambda,T}$.
In order to describe dust emission astronomers often use a modified graybody

$$S_\nu = K \; \nu^\beta \; B_{\nu,T_d} \; ,$$

where K is a constant, and β (whose value is between 1 and 2) is called the dust emissivity index. For such a modified graybody, in other words, $\varepsilon_\nu = K \; \nu^\beta$.

3 Object Spectrum

Sensitivity calculations can be performed with reference to theoretical spectra given in analytical form, to synthetic spectra derived from numerical simulations and fitted by some mathematical function, or to templates taken from the literature. Two common analytical forms are the flat spectrum

$$F_\lambda \equiv F_{\lambda,0} \quad [\mathrm{J \,/\, m\, m^2\, s}] \; ,$$

and the constant reduced brightness spectrum

$$\lambda \; F_\lambda \equiv F_0 \quad [\mathrm{J \,/\, m^2\, s}] \; .$$

When integrated over a wavelength interval centred on λ_0 and of width $\Delta\lambda$

$$\left[\lambda_0 - \frac{\Delta\lambda}{2}, \lambda_0 + \frac{\Delta\lambda}{2}\right] \; ,$$

these two spectra yield[2]

$$F_{\Delta\lambda} = \Delta\lambda \; F_{\lambda,0} \; ,$$

and

$$F_{\Delta\lambda} = \ln\left(\frac{\lambda_0 + \Delta\lambda/2}{\lambda_0 - \Delta\lambda/2}\right) \; F_0 \; .$$

To a first approximation, one can assume that the response of the system and the spectrum of the source are both flat, so as to integrate a constant

[2]Note, however, that in order to obtain the overall flux usable for scientific studies the spectrum of the source must be weighted by the transmittance of the optical telescope asembly (OTA) and by the detector quantum efficiency (DQE).

function over a given range. This will preclude the perception of possible details where the response and the spectrum show some structure such as rapid increases or decreases, but at the same time will allow simple computations.

4 Instrumental Background

In general, one must then know the geometry, temperature and emissivity of the different parts of the satellite in order to reliably simulate the instrumental background!

5 Sky Background

Measurements and units used for sky background (which from a dimensional point of view is of course a surface brightness) at different wavelengths are generally rather confusing.
Leinert *et al.* (1998) collect a lot of information about the different components of the sky background over a wide wavelength interval.

6 Signal-to-Noise Ratio Calculation

Under the assumption that an *observation* (or, equivalently, an *exposure*) of an object is made up of a certain number of *frames* (in order e.g. to reduce the total number of cosmic rays affecting the individual readouts and not to fill the potential wells of the detector' pixels, the conventions used in the following are as follows:

- t : single-frame exposure time [s]
- n_{fr} : number of frames composing an oservation [pure number]
- $T = n_{fr}\, t$: single-observation exposure time [s]
- object area : the angular size of the sky region over which a point source is spread, either due to diffraction effects or to the source being physically extended on the sky [solid angle - sr]
- n_p : number of pixels within the object area
- F : total electron counts during t [e^-]
- S : electron counts from the object during t [e^-]
- b_i : electron counts from the instrumental background per pixel during t [e^-]
- b_s : electron counts from the sky background per pixel during t [e^-]

- σ_f : standard error of generic estimated flux $f = F, S, b$ [e^-]
- r : total readnoise per pixel [e^- rms]
- SNR : signal-to-noise ratio in the measurement of the source brightness during an observation [pure number]

Under these assumptions, F can be written as

$$F = S + n_p\, b \ . \tag{8}$$

The physical process of the emission of photons from an astronomical object can be statistically described in terms of a Poisson distribution. The standard error in the measurement of F is then due to the intrinsic Poisson noise associated with F and to the readnoise. These two contributions sum quadratically yielding for the variance of F

$$\sigma_F^2 = F + n_p\, r^2 = S + n_p\, b + n_p\, r^2 \ . \tag{9}$$

Since the signal S from the object is estimated by subtraction from F of the sky background

$$S = F - n_p\, b \ , \tag{10}$$

the variance of S is

$$\sigma_S^2 = \sigma_F^2 + (n_p\, \sigma_b)^2 = S + n_p\, b + n_p\, r^2 + (n_p\, \sigma_b)^2 \ . \tag{11}$$

SNR is then

$$SNR = \sqrt{n_{obs}}\ \frac{S}{\sigma_S} = \frac{\sqrt{n_{obs}}\, S}{\sqrt{S + n_p\, b + n_p\, r^2 + (n_p\, \sigma_b)^2}} \ , \tag{12}$$

while σ_{mag} is

$$\sigma_{mag} = \frac{2.5 \log e}{\sqrt{n_{obs}}\ SNR} = \frac{2.5 \log e\ \sqrt{S + n_p\, b + n_p\, r^2 + (n_p\, \sigma_b)^2}}{\sqrt{n_{obs}}\ S} \ . \tag{13}$$

The overall SNR of a number n_{fr} of repeated observations of a given field is finally given by

$$SNR(T) = \sqrt{n_{fr}}\, SNR(t) \ .$$

A Angular Quantities

In astronomy, angular quantities are generally expressed in sexagesimal units. The main units of measure of plane and solid angles are the following:

$$1 \text{ degree} = 1 \text{ deg} \ ,$$

$$1 \text{ second of arc} = 1 \text{ arcsec} = 1 \text{ as} = \frac{1}{3600} \text{ deg} = 10^3 \text{ mas} = 10^6 \ \mu\text{as} ,$$

$$1 \text{ radian} = 1 \text{ rad} = \frac{180}{\pi} \text{ deg} = \frac{648000}{\pi} \text{ arcsec} ,$$

$$1 \text{ square degree} = 1 \text{ deg}^2 ,$$

$$1 \text{ steradian} = 1 \text{ sterad} = 1 \text{ sr} = \frac{32400}{\pi^2} \text{ deg}^2 = \frac{4.2 \cdot 10^{11}}{\pi^2} \text{ arcsec}^2 .$$

The whole sky spans a solid angle

$$\Omega_{sky} = 4\,\pi \text{ sterad} = \frac{129600}{\pi} \text{ deg}^2 = 41253 \text{ deg}^2 = \frac{1.68 \cdot 10^{12}}{\pi} \text{ arcsec}^2 ,$$

while the sky region where the absolute value of the Galactic (or Ecliptic, for that matter) latitude b is smaller than a given value ϕ measures

$$\Omega\,(|b| < \phi) = 4\,\pi\,\sin\phi \quad [\text{sr}] .$$

B Photometric Quantities

The nomenclature of photometric quantities in use in astronomical literature is far from standard and sometimes ambiguous. Here we therefore give a brief summary of the definitions and units of measure of these quantities as they are used in this study.

- The *Luminosity* L of a source is the energy radiated by the whole surface of the source per unit time, that is

$$L = \frac{\mathrm{d}E}{\mathrm{d}t} \ [\text{J / s}] . \tag{14}$$

- The *Brightness* F of a source is the energy radiated by the whole surface of the source per unit time per unit area (of the receiver), that is

$$F = \frac{\mathrm{d}L}{\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}A\,\mathrm{d}t} \ [\text{J / m}^2\,\text{s}] . \tag{15}$$

- The *Frequency Specific Brightness* F_ν of a source is the energy radiated by the whole surface of the source per unit time per unit area (of the receiver) per unit frequency interval, that is

$$F_\nu = \frac{\mathrm{d}F}{\mathrm{d}\nu} = \frac{\mathrm{d}L}{\mathrm{d}\nu\,\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}\nu\,\mathrm{d}A\,\mathrm{d}t} \ [\text{J / Hz m}^2\,\text{s}] . \tag{16}$$

  In astronomy, F_ν is generally expressed in Janskys, where

$$1 \text{ Jansky} = 1 \text{ Jy} = 10^{-26} \text{ J / s m}^2\,\text{Hz} .$$

Originally used by radio astronomers and named after one, while a rather perplexing unit itself (the meaning of the -26 exponent in particular having being extensively discussed in front of many a pint by discouraged early postgraduates), the Jansky is also important in that it provides the foundation for a magnitude system which, given the idiosincracies of most practitioning astronomers, is about as absolute as it gets, the socalled AB magnitude system. This is defined, at ANY wavelength, by setting $m_{AB} = 0$ for a source of $F_\nu = 3631$ Jy. This tranlates into the following conversions

$$F_{\nu\,[\mathrm{Jy}]} = 3631 \cdot 10^{-0.4\, m_{AB}} \ ,$$
$$m_{AB} = -2.5 \cdot \log F_{\nu\,[\mathrm{Jy}]} + 8.9 \ .$$

Conversely, the suitably-named Vega magnitudes are defined so that the magnitude of Vega is zero in all bands. Somewhat confusingly, absolute fluxes of Vega (and secondary calibrators) are measured with increasing accuracy and thus the very definition of Vega magnitudes changes in time. Having said that, estimates for the conversion factor defined by

$$m_{AB} = m_{Vega} + conv \tag{17}$$

are given in Table 2. Clearly, *conv* is simply the magnitude of the Vega star in the AB system (in a given band), so that

$$F_{zp\,[\mathrm{Vega}]} = F_{zp\,[AB]} \cdot 10^{-0.4\, conv} \ ,$$

or equivalently

$$conv = 2.5\ \log(F_{zp\,[AB]}/F_{zp\,[Vega]}) \ ,$$

where

$$\text{where} \quad F_{zp\,[AB]} = 3631 \text{ Jy} \ .$$

The closely related *Wavelength Specific Brightness* F_λ of a source is the energy radiated by the whole surface of the source per unit time per unit area (of the receiver) per unit wavelength interval, that is

$$F_\lambda = \frac{\mathrm{d}F}{\mathrm{d}\lambda} = \frac{\mathrm{d}L}{\mathrm{d}\lambda\,\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}\lambda\,\mathrm{d}A\,\mathrm{d}t}\ [\mathrm{J}\,/\,\mathrm{m\,m^2\,s}] \ . \tag{18}$$

The *Frequency Specific Reduced Brightness* $F_{\nu,red}$ of a source is the

Table 2: AB vs Vega Magnitudes. Band, *conv* factors defined in Equation 17, Vega zero-points (or zero-magnitude fluxes, in Jy) and source.

band	*conv*	F_{zp}	source
SDSS u	0.927	1545	Hewett et al. 2006
SDSS g	-0.103	3991	Hewett et al. 2006
SDSS r	0.146	3174	Hewett et al. 2006
SDSS i	0.366	2593	Hewett et al. 2006
SDSS z	0.533	2222	Hewett et al. 2006
2MASS J	0.89	1594	2MASS Expl. Supp. 2006
2MASS H	1.37	1024	2MASS Expl. Supp. 2006
2MASS K_s	1.84	666.7	2MASS Expl. Supp. 2006
UKIDSS Z	0.528	2232	Hewett et al. 2006
UKIDSS Y	0.634	2026	Hewett et al. 2006
UKIDSS J	0.938	1530	Hewett et al. 2006
UKIDSS H	1.379	1019	Hewett et al. 2006
UKIDSS K	1.900	631	Hewett et al. 2006
IRAC-1	2.78	280.9	IRAC Data HandBook 3.0 2006
IRAC-2	3.26	179.7	IRAC Data HandBook 3.0 2006
IRAC-3	3.75	115.0	IRAC Data HandBook 3.0 2006
IRAC-4	4.38	64.1	IRAC Data HandBook 3.0 2006
MIPS-24	6.77	7.14	MIPS Data HandBook 3.3.1 2008
MIPS-70	9.18	0.775	MIPS Data HandBook 3.3.1 2008
MIPS-160	10.90	0.159	MIPS Data HandBook 3.3.1 2008

Table 3: AB vs Vega Magnitudes in Johnson-Counsins-Glass System. Band, Effective Wavelength and Width in nm, *conv* factors defined in Equation 17 and Vega zero-points (or zero-magnitude fluxes, in Jy). From `http://www.euro-vo.org/internal/Avo/WorkPackageTwoTwo/mag_to_flux_conversions` and to be taken with a bit of skepticism.

band	λ_{eff}	$\Delta\lambda$	*conv*	F_{zp}
U	367	66	0.767933	1790
B	436	94	-0.122051	4063
V	545	85	-0.001494	3636
R	638	160	0.184344	3064
I	797	149	0.442323	2416
J	1220	213	0.897256	1589
H	1630	307	1.37857	1020
K	2190	390	1.88462	640
L	3450	472	2.76295	285
M	4750	460	3.43126	154

product of its Frequency Specific Brightness and the frequency at which it is measured, namely

$$F_{\nu,red} = \nu\, F_\nu \ [\mathrm{J} \ / \ \mathrm{m}^2 \ \mathrm{s}] \ , \tag{19}$$

whereas the *Wavelength Specific Reduced Brightness* $F_{\lambda,red}$ of a source is the product of its Wavelength Specific Brightness and the wavelength at which it is measured, namely

$$F_{\lambda,red} = \lambda\, F_\lambda \ [\mathrm{J} \ / \ \mathrm{m}^2 \ \mathrm{s}] \ . \tag{20}$$

The fact that these two quantities are expressed in the same units is not a mere coincidence, but rather to their being the same physical quantity, expressing the source brightness contained in a given (logarithmic) spectral range. In other words, if we plot $F_{\nu,red} = F_{\lambda,red}$ using a logarithmic scale along the abscissae, the brightness emitted over different wavelength ranges can be directly compared by comparing the areas under the relevant regions of the curve, making it a powerful tool in graphically illustrating the emission processes of astronomical sources.

- Most galaxies, unlike most stars, are resolved objects, so that in addition to measuring their total energy flux, we can in principle measure the energy flux per unit solid angle of the source coming from different regions. The *Surface Brightness* Σ of a region of a diffuse source is the energy radiated by the region per unit time, per unit area (of the receiver) and per unit solid angle (of the source), that is

$$\Sigma = \frac{\mathrm{d}F}{\mathrm{d}\Omega} = \frac{\mathrm{d}L}{\mathrm{d}\Omega\,\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}\Omega\,\mathrm{d}A\,\mathrm{d}t} \ [\mathrm{J} \ / \ \mathrm{sr} \ \mathrm{m}^2 \ \mathrm{s}] \ . \tag{21}$$

- The *Frequency Specific Surface Brightness* Σ_ν of a region of a diffuse source is the energy radiated by the region per unit time, per unit area (of the receiver) per unit solid angle (of the source) per unit frequency interval, that is

$$\Sigma_\nu = \frac{\mathrm{d}F}{\mathrm{d}\nu\,\mathrm{d}\Omega} = \frac{\mathrm{d}L}{\mathrm{d}\nu\,\mathrm{d}\Omega\,\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}\nu\,\mathrm{d}\Omega\,\mathrm{d}A\,\mathrm{d}t} \ [\mathrm{J} \ / \ \mathrm{Hz} \ \mathrm{sr} \ \mathrm{m}^2 \ \mathrm{s}] \ . \tag{22}$$

The closely related *Wavelength Specific Surface Brightness* Σ_λ of a region of a diffuse source is the energy radiated by the region per unit time, per unit area (of the receiver) per unit solid angle (of the source) per unit wavelength interval, that is

$$\Sigma_\lambda = \frac{\mathrm{d}F}{\mathrm{d}\lambda\,\mathrm{d}\Omega} = \frac{\mathrm{d}L}{\mathrm{d}\lambda\,\mathrm{d}\Omega\,\mathrm{d}A} = \frac{\mathrm{d}E}{\mathrm{d}\lambda\,\mathrm{d}\Omega\,\mathrm{d}A\,\mathrm{d}t} \ [\mathrm{J} \ / \ \mathrm{m} \ \mathrm{sr} \ \mathrm{m}^2 \ \mathrm{s}] \ . \tag{23}$$

- When Luminosities rather than Brightnesses are being measured, L_ν and L_λ are defined rather than F_ν and F_λ.

Note that since

$$\mathrm{d}\nu = \mathrm{d}\left(\frac{c}{\lambda}\right) = \frac{c}{\lambda^2}\,\mathrm{d}\lambda \quad \Longleftrightarrow \quad \frac{\mathrm{d}\nu}{\nu} = \frac{\mathrm{d}\lambda}{\lambda}\ , \tag{24}$$

the two following relations hold

$$F_\nu = \frac{\lambda^2}{c}\,F_\lambda \quad \Longleftrightarrow \quad \frac{F_\nu}{\nu} = \frac{\lambda}{\lambda}\ , \tag{25}$$

$$\Sigma_\nu = \frac{\lambda^2}{c}\,\Sigma_\lambda \quad \Longleftrightarrow \quad \frac{\Sigma_\nu}{\nu} = \frac{\Sigma_\lambda}{\lambda}\ . \tag{26}$$

Astronomers, however, generally express brightness and surface brightness in logarithmic units, i.e. in magnitudes (mag) and magnitudes per square second of arc (mag/arcsec^2), respectively. To define a magnitude scale, one has to arbitrarily choose a reference brightness F_{zp}, and the corresponding reference surface brightness Σ_{zp} of F_{zp} per square second of arc. The brightness of a source expressed in magnitudes is then

$$m = -2.5\ \log\frac{F}{F_{zp}}\ [\mathrm{mag}]\ , \tag{27}$$

while the surface brightness of a region of a diffuse source in magnitudes per square second of arc is

$$\mu = -2.5\ \log\frac{\Sigma}{\Sigma_{zp}}\ [\mathrm{mag/arcsec^2}]\ . \tag{28}$$

F_{zp} is called the zero-point of the adopted magnitude scale since $m = 0$ for $F = F_{zp}$ (and thus $\mu = 0$ for $\Sigma = \Sigma_{zp}$).
Note that these definitions equally apply to bolometric measurements and to measurements in a given photometric band. One then simply has to take into account only the radiation within a given wavelength range weighted by the profile of the photometric band.
Note also that the sky background is often expressed in different units, such as those described by Leinert *et al.* (1998).

C Numerical Constants

$$\pi = 3.141... \quad e = 2.718... \quad \gamma = 0.577...$$

$$\log e = 0.434... \quad \ln 10 = 2.302...$$

$$\log e\ \ln 10 = 1$$

See also Table 4.

log(01)	0.000000	dex(0.1)	1.25893
log(02)	0.301030	dex(0.2)	1.58489
log(03)	0.477121	dex(0.3)	1.99526
log(04)	0.602060	dex(0.4)	2.51189
log(05)	0.698970	dex(0.5)	3.16228
log(06)	0.778151	dex(0.6)	3.98107
log(07)	0.845098	dex(0.7)	5.01187
log(08)	0.903090	dex(0.8)	6.30957
log(09)	0.954243	dex(0.9)	7.94328
log(10)	1.000000	dex(1.0)	10.0000

Table 4: Some useful numerical constants

D Miscellaneous "Numbers"

Two units which are often used to express surface brightness and sky background measurements are related by

$$1\ \mathrm{MJy/sr} = \frac{\pi^2}{0.419904}\ \mu\mathrm{Jy/arcsec}^2 \simeq 23.5\ \mu\mathrm{Jy/arcsec}^2\ . \qquad (29)$$

Given the privileged role of the Sun in shaping our view of the Universe, the *Solar Luminosity, Mass and Radius*, i.e. the luminosity, mass and radius units often used to express the luminosities, masses and radii of stars and/or galaxies, are equal to the (currently accepted) solar values, i.e.

$$L_\odot = 3.8478 \cdot 10^{26}\ \mathrm{J/s} = 3.8478 \cdot 10^{33}\ \mathrm{erg/s} \qquad (30)$$

$$M_\odot = 1.9891 \cdot 10^{30}\ \mathrm{kg} = 1.9891 \cdot 10^{33}\ \mathrm{g} \qquad (31)$$

$$R_\odot = 6.960 \cdot 10^{8}\ \mathrm{m} = 6.960 \cdot 10^{10}\ \mathrm{cm} \qquad (32)$$

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